Astrophysics & Intro. Astrophysics I **NAME:**
HW #2
Due: Wed. Oct. 7

1. An "astronomer-friendly" version of the Jeans Mass is $M_J \approx 10^5 \frac{T^{3/2}}{\sqrt{n}} M_{Sun}$, where n is the number density of gas molecules *per* m^3.

a. For a "typical" mean density of $n = 50\ cm^{-3}$ and temperature $T = 20\ K$, calculate the Jeans Mass in solar masses. *(2 point)*

b. What sort of "object" would this sort of mass correspond to? (i.e. massive star, star cluster, galaxy, or ??). *(1 point)*

c. Assume that the cloud that collapsed to form the Sun originally had a mass of 2 M_{Sun}. If it had an original temperature of 20 K, what would the lower limit be on its density (list the density on number per m^3 and per cm^3). *(2 point)*

d. Assuming that the collapse takes place on a time scale that is approximately that of a free-fall collapse, how long would it take for the cloud that formed the sun to do so (give your answer in *seconds* **and** in *years*, and assume that the gas is *molecular* hydrogen). *(2 point)*

(Note: a number to remember: there are 3.155×10^7 sec per year or about "$\pi \times 10^7$ sec/yr")

2. Use the physical conditions for the Standard Solar Model tabulated *(in cgs units!)* in the lecture notes for $M_r / M_{Sun} = 0.0000298$ (the top line of the table) to calculate the following quantities:

a. the pressure due to radiation *(1 point)*

b. the pressure due to (non-relativistic) electron degeneracy, using $\mu_e = \dfrac{2}{1+X}$. *(2 point)*

c. What fraction of the total pressure do these two components contribute individually? *(1 point)*

3. Using the figure for the mass-luminosity relation for main sequence stars that I have in my class notes (last slide of Ch. 2), derive an approximate analytic expression for the mean M-L relation for stars with masses *smaller* than that of the Sun, similar to the one in my notes. *(1 point)*

Use this relation to estimate the lifetimes of the lowest-mass stars in the figure, and compare this to both the estimated lifetime of the Sun, and to the present age of the universe (about 14 Gyrs = 14×10^9 yrs). *Remember that these stars are fully convective throughout their volumes. (2 points)*

NOTE: It is easier to work this problem out in "astro units" like solar masses, etc. This leads to fewer "opportunities" to get something wrong. There is also NO NEED to go back and re-derive the stuff that is already derived in the class notes. For example, you already have been given the fact that H-burning is 0.7% "efficient" in converting mass into energy. Going through that again takes extra time. It also does nothing to impress me! I assume you know this.

GRADUATE STUDENTS – This is Problem #3 from Ch. 3 of the textbook.

Because of the destabilizing influence of radiation pressure (see Problem 1 of Chapter 3 in the textbook) the most massive stars that can form are those in which the radiation pressure and the non-relativistic kinetic pressure are approximately equal. Estimate the mass of the most massive stars as follows.

a. Assume that the gravitational binding energy of a star of mass M and radius R is $|E_{gr}| \sim GM^2/R$. Use the virial theorem (Eq. 3.22)

$$\overline{P} = -\frac{1}{3}\frac{E_{gr}}{V}$$

to show that

$$P \sim \left(\frac{4\pi}{3^4}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

where ρ is the typical density. *(2 points)*

b. Show that if the radiation pressure, $P_{rad} = \frac{1}{3}aT^4$, equals the kinetic pressure, then the total pressure is

$$P_{TOTAL} = 2\left(\frac{3}{a}\right)^{1/3}\left(\frac{k\rho}{\overline{m}}\right)^{4/3} \text{ where } \overline{m} = \mu m_H$$ *(2 points)*

c. Equate the expressions for the pressure in (a) and (b), to obtain an expression for the maximal mass of a star, and show that its mass is $M \sim 110 M_{\odot}$. *(2 points)*