Astrophysics & Intro. Astrophysics I **NAME:**
HW #5
Due: Monday November 9

1. In the Solar neighborhood, the Milky Way has a flat rotation curve, with $v(r) = v_c$ where v_c is a constant, implying a mass density profile $\rho(r) \sim r^{-2}$ (see Eq. 6.18 in your textbook).

a. Assume there is a cutoff radius R beyond where the mass density is zero. Prove that the velocity of escape from the galaxy from any radius $r < R$ is

$$v_e^2 = 2v_c^2\left(1 + \ln\frac{R}{r}\right)$$

Hint: you will need to do the integral in two parts. *(4 points)*

b. The largest velocity measured for any star in the Solar neighborhood, at $r = 8\ kpc$, is $440\ km\ s^{-1}$. If this star is still bound to the galaxy, show that the cutoff radius $R > 22\ kpc$ and that the mass of the galaxy interior to R, $M(R) > 2.4x10^{11} M_{\odot}$. The solar velocity about the galaxy is $v_c = 220\ km\ s^{-1}$ and the mass interior to the Sun's orbit is $1.8x10^{44}\ g \approx 0.9x10^{11} M_{\odot}$ (Eq. 6.5 in the textbook). *(2 points)*

2. NGC 2639 is an Sa galaxy with a measured maximum rotational velocity of 324 km/s and an apparent magnitude of B=12.22 mag.

a. Using the graphical representation of the Tully-Fisher relation from the notes, estimate the absolute magnitude in the B band (M_B). *(1 point)*

b. Determine the distance to NGC 2639. *(1 point)*

c. It has been discovered that the radius outward in the disk where the surface brightness of such galaxies drops to 25 mag arcsec^{-2} in the B band can be well represented by

$$\log_{10} R_{25} = -0.249 M_B - 4.00$$

where R_{25} is in units of kiloparsecs. What is R_{25} for NGC 2639? *(1 point)*

d. Finally, using the fact that the mass interior to R is given by $M = \frac{V_{\max}^2 R}{G}$, calculate the mass interior to R_{25} for NGC 2639. Note: the formula here is in standard physics units, so the mass will be in kg or g, whichever way you do the problem. **Give your final answer in *solar masses*.** *(1 point)*

3. Dynamical friction, which is a force, must be made up of some combination of it mass (GM), its velocity v_M, and mass density ρ.

$$f_d = C(GM)^{\alpha}(v_M)^{\beta}\rho^{\gamma}$$

Here C is a dimensionless constant. Calculate what the exponents α, β, and γ must be in order for the units (length, mass, time) to be equal on each side of the equation. To do this, simply substitute the units appropriate for each term in the equation above. This will yield 3 algebraic equations to solve for the 3 unknowns. **Because the final answer is given in the class notes, you MUST show your work!** *This is an example of dimensional analysis, and doing this lets you guess at the form an equation must take*. (I love putting one of these on the graduate student's Qualifying Exam!) Lord Rayleigh used this to determined the functional form of the type of light scattering that bears his name. *(2 points)*

4. On Page 4 of the Ch. 20 notes (AGN-I) I show a "composite" QSO spectrum, plotted in λF_λ units (W m^{-2}).

a. If this spectrum is that produced by the accretion disk of a supermassive black hole, which of the models shown on Page 7 of the Ch. 21 notes (AGN-II) is closest to the one required to match the maximum brightness of the composite spectrum? Note: the model is plotted in units of $\nu F_\nu = \lambda F_\lambda$. *(2 points)*

b. Is the maximum temperature for this model approximately equal (to within a factor of 2 or so) of what you would get by applying Wien's Law to the "composite" QSO spectrum? *(1 point)*

c. For simplicity, let us suppose that the temperature in the disk is given by $T(r) = T_{in}\left(\frac{r_{in}}{r}\right)^{3/4}$,

$r_{in} = 6R_G = 3R_{Schwarzschild} = \frac{6GM}{c^2}$, and let $T_{in} = 50{,}000K$.

Calculate the total luminosity of the disk (remember that it has 2 faces!) assuming that the locally-emitted flux at any location in the disk is given by the Stefan-Boltzmann Law, and that the outer radius of the disk is at 100 AU. Give your answer in units of solar luminosities $L_{\odot}$ *(5 points)*