11 – Local Thermodynamic Equilibrium & Radiative Equilibrium

Thermal Equilibrium

Due to the MFPs being small for ions/electrons and large for photons:

Ion-ion (electron) interactions are very close to TE
Ion-photon interactions deviate from TE

But the locally-generated S_ν depends heavily on the ion-ion interactions, and so it will often not deviate drastically from the TE of its locality.

For a stellar photosphere, LTE is the assumption that:

The Kinetic Energy distribution of matter particles
The Excitation distribution of matter particles
The Ionization distribution of matter particles

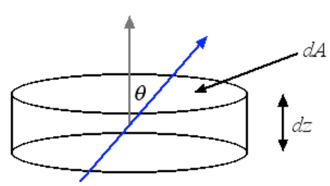
are all given by the equilibrium relations using the local value of T (from KE). That is, $T_{KE} = T_{exc} = T_{ion}$. This simplifies matters somewhat, and also gives:

$$j_\nu^{thermal} = \kappa_\nu B_\nu$$

Scattering Equilibrium

This condition states that:
Incident energy lost due to scattering in a volume element

= Energy scattered out of that volume



$$\left(dE_\nu\right)_{\substack{\text{emitted by}\\ \text{scattering}}} = j_\nu^s \rho \overbrace{dAdz}^{dV} d\omega dt d\nu$$

$$\left(dE_\nu\right)_{\substack{\text{incident}\\ \text{which is}\\ \text{scattered}}} = \sigma_\nu \rho I_\nu dAdz d\omega dt d\nu$$

We will assume that $\left\{\begin{matrix} \sigma_\nu \neq \sigma_\nu(\omega) \\ j_\nu^s \neq j_\nu^s(\omega) \end{matrix}\right\}$, i.e. they are independent of direction

This gives: $j_\nu^s = \sigma_\nu J_\nu$

General Source Function

$$S_\nu = \frac{j_\nu}{k_\nu} = \frac{\left(j_\nu^{th} + j_\nu^{scatt}\right)}{\kappa_\nu + \sigma_\nu} = \frac{\kappa_\nu B_\nu + \sigma_\nu J_\nu}{\kappa_\nu + \sigma_\nu}$$ **LTE Source Function**

Radiative Equilibrium

The TOTAL INTEGRATED energy emitted by thermal emission and scattering by a volume is equal to the TOTAL INTEGRATED amount of energy it absorbs by true emission and scattering (not necessarily true for each ν individually). This is a somewhat less restrictive condition that what we just looked at. We will use the same starting point:

$$\left(dE_\nu\right)_{emitted} = j_\nu \rho \overbrace{dAdz}^{dV} d\omega dt d\nu$$

$$\left(dE_\nu\right)_{absorbed} = k_\nu \rho I_\nu dAdz d\omega dt d\nu$$

Now, let's integrate over ν and ω

$$\int_0^\infty \oint j_\nu d\omega d\nu = \int_0^\infty \oint k_\nu I_\nu d\omega d\nu$$

$$or \quad \int_0^\infty j_\nu d\nu = \int_0^\infty k_\nu J_\nu d\nu$$

Furthermore (with just a little bit of LTE),

$$\int_0^\infty \left(j_\nu^{th} + j_\nu^{sc}\right) d\nu = \int_0^\infty \left(\kappa_\nu + \sigma_\nu\right) J_\nu d\nu$$

$$\int_0^\infty \kappa_\nu B_\nu d\nu + \int_0^\infty \sigma_\nu J_\nu d\nu = \int_0^\infty \kappa_\nu J_\nu d\nu + \int_0^\infty \sigma_\nu J_\nu d\nu \quad \textbf{RADIATIVE EQUILIBRIUM}$$

$$so \quad \int_0^\infty \kappa_\nu B_\nu d\nu = \int_0^\infty \kappa_\nu J_\nu d\nu$$

Radiative equilibrium has another consequence. Using the Equation of Transfer:

$$\oint \cos\theta \frac{dI_\nu}{dz} d\omega = \oint j_\nu \rho d\omega - \oint k_\nu \rho I_\nu d\omega$$

$$\frac{d}{dz}\oint \cos\theta \, I_\nu d\omega = \frac{d\Im_\nu}{dz} = \rho\left[\oint_\omega j_\nu d\omega - \oint_\omega k_\nu I_\nu d\omega\right]$$

Now, integrate over all ν,

$$\int_0^\infty \frac{d\Im}{dz} = \rho \underbrace{\left[\oint_\omega \int_0^\infty j_\nu d\omega d\nu - \oint_\omega \int_0^\infty k_\nu I_\nu d\omega d\nu\right]}_{=0 \quad in\ RAD\ EQ.}$$

$$\frac{d\Im}{dz} = 0 \quad \Rightarrow \quad \Im = constant \quad \textbf{RADIATIVE EQUILIBRIUM}$$