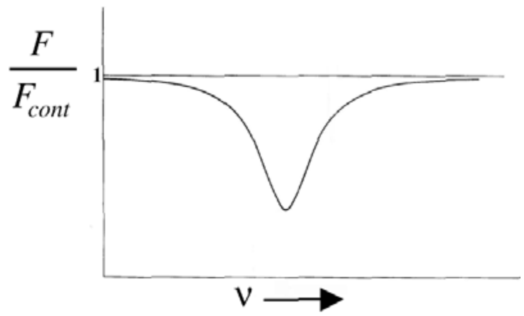
14 – LINES I

Let $\ell_\nu = \ell_\nu^{thermal} + \ell_\nu^{scattering}$ be the extinction coefficient in lines. Then the Eq of T becomes

$$\frac{\mu}{\rho}\frac{dI_\nu}{dz} = j_\nu - k_\nu I_\nu = \sigma_\nu J_\nu + \kappa_\nu B_\nu + \ell_\nu^{sc} J_\nu + \ell_\nu^{th} B_\nu - \left(\sigma_\nu + \kappa_\nu + \ell_\nu\right) I_\nu$$

After considerable (and unglamorous) manipulation, it can be shown that at the surface, the emergent flux and specific intensity can be described approximately by:

$$\frac{F_\nu(0)}{F_c(0)} = \frac{\left(\frac{1}{1+\eta_\nu} + \frac{4}{9}\sqrt{3\lambda_\nu}\right)\left(1+\sqrt{1-\rho}\right)}{\left(1+\frac{4}{9}\sqrt{3(1-\rho)}\right)\left(1+\sqrt{\lambda_\nu}\right)}$$



$$\frac{I_\nu(0,\mu)}{I_c(0,\mu)} = \frac{\frac{4}{9} + \frac{\mu}{1+\eta_\nu} + \frac{\left(\frac{1}{\sqrt{3}(1+\eta_\nu)} - \frac{4}{9}\right)\left(1-\lambda_\nu\right)}{\left(1+\sqrt{\lambda_\nu}\right)\left(1+\sqrt{3\lambda_\nu}\,\mu\right)}}{\frac{4}{9} + \mu + \frac{\left(\frac{1}{\sqrt{3}} - \frac{4}{9}\right)\rho}{\left(1+\sqrt{1-\rho}\right)\left(1+\sqrt{3(1-\rho)}\,\mu\right)}}$$

Where the subscript "c" refers to the continuum, and

$\eta_\nu = \frac{\ell_\nu}{\kappa + \sigma}$, $\rho = \frac{\sigma}{\sigma + \kappa}$, $\lambda_\nu = \frac{1-\rho+\varepsilon\eta_\nu}{1+\eta_\nu}$ and ε is the fraction of upward transitions that lead to "thermal" (as opposed to "scattering") downward transitions:

$$\ell_\nu^{th} = \varepsilon \ell_\nu \qquad \ell_\nu^{sc} = (1-\varepsilon)\ell_\nu$$

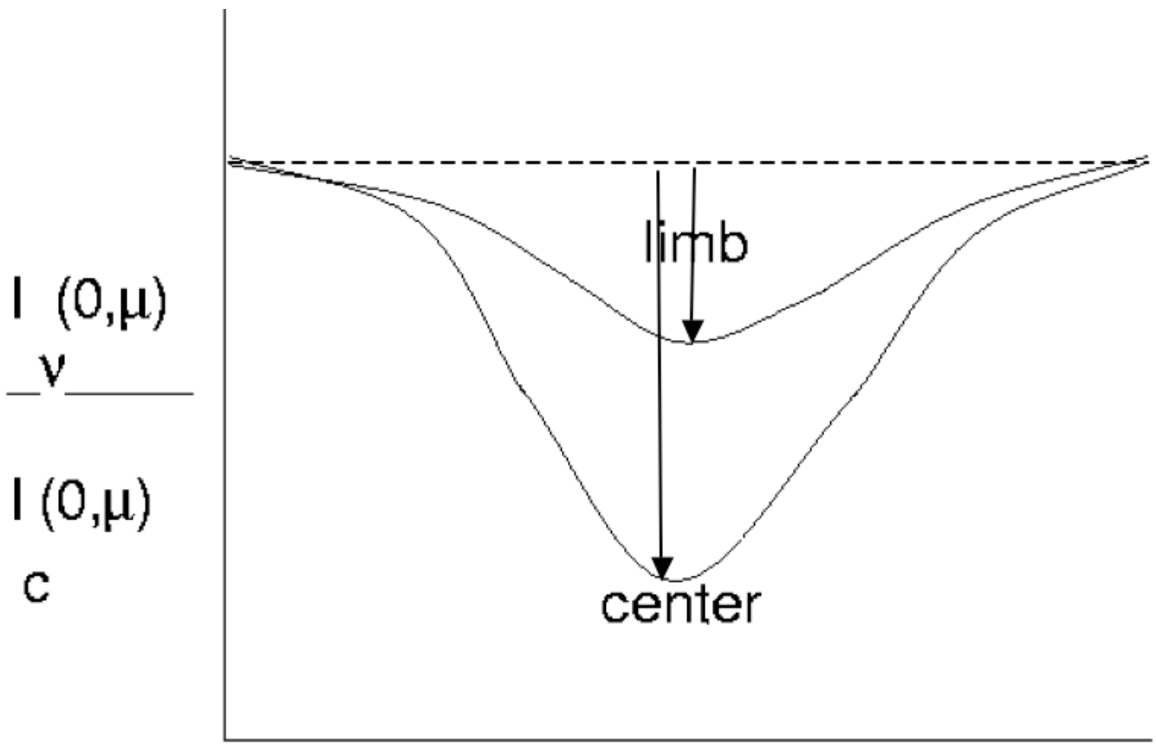
(To get to this point requires the use of the Eddington Approximation $J_\nu = 3K_\nu$ and assumes you can represent the Planck function as $B_\nu(T) = a_\nu + b_\nu \tau_\nu$.

There are 4 "ideal" cases:

		CONTINUUM	
		No Scattering $(\rho = 0)$	*Pure Scattering* $(\rho = 1)$
LINE	*No Scattering* $(\varepsilon = 1)$	Case B $\begin{Bmatrix}\rho = 0 \\ \varepsilon = 1\end{Bmatrix} \Rightarrow \lambda_\nu = 1$	Case D $\begin{Bmatrix}\rho = 1 \\ \varepsilon = 1\end{Bmatrix} \Rightarrow \lambda_\nu = \frac{\eta_\nu}{1+\eta_\nu}$
	Pure Scattering $(\varepsilon = 0)$	Case A $\begin{Bmatrix}\rho = 0 \\ \varepsilon = 0\end{Bmatrix} \Rightarrow \lambda_\nu = \frac{1}{1+\eta_\nu}$	Case C $\begin{Bmatrix}\rho = 1 \\ \varepsilon = 0\end{Bmatrix} \Rightarrow \lambda_\nu = 0$

Case A – Thermal Continuum and Scattering Line

$\eta_\nu \setminus \mu$	*1*	$\frac{I_\nu(0,\mu)}{I_c(0,\mu)}$ *1/2*	*0*	$\frac{F_\nu(0)}{F_c(0)}$
0	1.00	1.00	1.00	1.00
1/10	0.94	0.95	1.01	0.95
1	0.64	0.71	0.90	0.69
10	0.25	0.29	0.38	0.28
100	0.08	0.09	0.11	0.09
∞	0	0	0	0

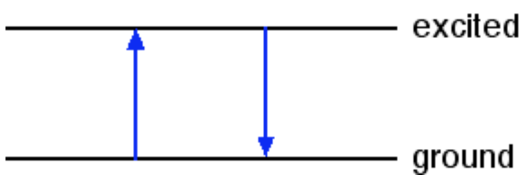


Case B – Thermal Continuum and Thermal Line

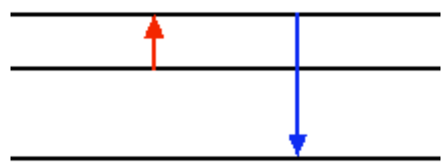
$\eta_\nu \backslash \mu$	*1*	$\frac{I_\nu(0,\mu)}{I_c(0,\mu)}$ *1/2*	*0*	$\frac{F_\nu(0)}{F_c(0)}$
0	1.00	1.00	1.00	1.00
1/10	0.94	0.95	1.00	0.95
1	0.65	0.74	1.00	0.72
10	0.37	0.52	1.00	0.49
100	0.31	0.48	1.00	0.44
∞	0.31	0.47	1.00	0.43

Cases A and B $\Rightarrow$ Continuum is Thermal

Case A Lines
Resonant lines from ground state



Case B Lines
Weaker, subordinate lines from excited states



Example: The Na I "D" lines are resonant lines, and are very deep in the solar spectrum.

Case C Scattering Continuum $(\rho = 1)$ and Scattering Lines $(\varepsilon = 0) \Rightarrow \lambda_\nu = 0$

$$\left\{ \begin{array}{c} \dfrac{I_\nu(0,\mu)}{I_c(0,\mu)} \\ \dfrac{F_\nu(0)}{F_c(0)} \end{array} \right\} \Rightarrow = \frac{1}{1+\eta_\nu}$$

Get a net line due to greater scattering in the line than in the continuum, but deep in the atmosphere the model must break down. It also contains some self-inconsistencies, since to get this it had to be assumed that $J_\nu = B_\nu \; as \; \tau_\nu \to \infty$ and $B \propto T^4$ (i.e. *thermal* conditions) to get to this point.

Case D Scattering Continuum $(\rho = 1)$ and Thermal Lines $(\varepsilon = 1) \Rightarrow \lambda_\nu = \dfrac{\eta_\nu}{1+\eta_\nu}$

$\eta_\nu \backslash \mu$	*1*	$\dfrac{I_\nu(0,\mu)}{I_c(0,\mu)}$ *1/2*	*0*	$\dfrac{F_\nu(0)}{F_c(0)}$
0	1.00	1.00	1.00	1.00
1/10	0.88		0.87	0.88
1	0.59		0.69	0.61
10	0.34		0.74	0.42
100	0.29		0.77	0.39
∞	0.28		0.77	0.38

Here, the line can actually go into emission at the limb. It depends on the ratio a/b in $B_\nu(T) = a_\nu + b_\nu \tau_\nu$.