THE LANE-EMDEN FUNCTION $\theta_{3.25}$

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The Lane-Emden function $\theta_{3.25}$ is of considerable interest in the study of stellar models. Thus a stellar model with a uniform distribution of the energy sources ($\eta = 1$), of small mass ($1 - \beta \sim 0$) and on Kramer's law of opacity ($\kappa = \kappa_0 \rho T^{-3.5}$), will be described by $\theta_{3.25}$. Further, if stars have an inner core in which the energy sources are uniformly distributed, they will be described by a point-source envelope fitted onto a polytropic core of index 3.25.[1] For these and similar purposes it has been found necessary to integrate the function $\theta_{3.25}$, which has not been available so far. Table 1 gives the required function and certain other auxiliary functions which are of importance in the theory.

The function $\theta_{3.25}$ has its first zero at $\xi = \xi_1$, where

$$\xi_1 = 8.01894\,. \tag{1}$$

At $\xi = \xi_1$ we have

$$\left.\begin{aligned} -\theta' &= 0.030322\,, \\ -\xi^2\theta' &= 1.94980\,, \\ \frac{\rho_c}{\bar{\rho}} &= 88.153\,. \end{aligned}\right\} \tag{2}$$

Applications of this solution will also be found in the author's monograph, *An Introduction to the Study of Stellar Structure*.[2]

Regarding the accuracy of the table, it may be mentioned that an error of more than two (and in extreme cases three) units in the last figure retained is not expected.

[1] This model is being studied by the writer, and the results of this study will be published in due course.

[2] Chicago: University of Chicago Press (in press).

TABLE 1

ξ	θ	$-\theta'$	$\theta^{3.25}$	$\theta^{4.25}$	$-\xi^2\theta'$	$-\frac{\xi}{3\theta'}$
0....	1.000000	0	1.00000	1.00000	0	1.0000
0.1..	0.998336	.033225	0.99460	0.99295	0.0003323	1.0033
0.2..	0.993376	.065809	0.97863	0.97215	0.0026323	1.0130
0.3..	0.985216	.097140	0.95275	0.93866	0.0087426	1.0294
0.4..	0.974009	.126668	0.91797	0.89411	0.020267	1.0526
0.5..	0.959958	.153926	0.87563	0.84057	0.038482	1.0828
0.6..	0.943311	.17855	0.82724	0.78034	0.064277	1.1201
0.7..	0.924345	.20027	0.77439	0.71581	0.098134	1.1651
0.8..	0.903358	.21895	0.71870	0.64924	0.14013	1.2179
0.9..	0.880658	.23454	0.66164	0.58268	0.18998	1.2791
1.0..	0.856553	.24707	0.60458	0.51785	0.24707	1.3491
1.1..	0.831341	.25668	0.54863	0.45610	0.31058	1.4285
1.2..	0.805309	.26353	0.49474	0.39842	0.37948	1.5179
1.3..	0.778720	.26786	0.44360	0.34544	0.45269	1.6178
1.4..	0.751813	.26992	0.39569	0.29749	0.52904	1.7289
1.5..	0.724803	.26997	0.35133	0.25465	0.60743	1.8521
1.6..	0.697877	.26828	0.31066	0.21680	0.68680	1.9880
1.7..	0.671196	.26512	0.27369	0.18370	0.76619	2.1374
1.8..	0.644895	.26072	0.24035	0.15500	0.84473	2.3013
1.9..	0.619086	.25532	0.21047	0.13030	0.92170	2.4806
2.0..	0.593858	.24913	0.18385	0.10918	0.99651	2.6760
2.1..	0.569281	.24232	0.16026	0.091230	1.06864	2.8887
2.2..	0.545408	.23507	0.13943	0.076044	1.13774	3.1196
2.3..	0.522277	.22751	0.12111	0.063252	1.20353	3.3698
2.4..	0.499912	.21976	0.10505	0.052517	1.26583	3.6403
2.5..	0.478327	.21193	0.091014	0.043534	1.32453	3.9322
2.6..	0.457527	.20408	0.078769	0.036039	1.37959	4.2467
2.7..	0.437509	.19630	0.068109	0.029799	1.43103	4.5848
2.8..	0.418263	.18864	0.058845	0.024613	1.47893	4.9477
2.9..	0.399776	.18114	0.050805	0.020311	1.52335	5.3367
3.0..	0.382030	.17382	0.043834	0.016746	1.56442	5.7529
3.1..	0.365004	.16673	0.037798	0.013796	1.60229	6.1976
3.2..	0.348675	.15988	0.032574	0.011358	1.63713	6.6718
3.3..	0.333020	.15327	0.028056	0.009343	1.66906	7.1771
3.4..	0.318014	.14691	0.024152	0.007681	1.69828	7.7145
3.5..	0.303630	.14081	0.020779	0.006309	1.72496	8.2852
3.6..	0.289842	.13497	0.017866	0.005178	1.74925	8.8907
3.7..	0.276627	.12939	0.015352	0.004247	1.77131	9.5321
3.8..	0.263956	.12405	0.013182	0.003479	1.79134	10.211
3.9..	0.251808	.11896	0.011310	0.002848	1.80944	10.928
4.0..	0.240156	.11411	0.009696	0.002329	1.82578	11.685
4.1..	0.228978	.10949	0.008305	0.001902	1.84051	12.482
4.2..	0.218251	.10509	0.007106	0.001551	1.85377	13.322
4.3..	0.207953	.10090	0.006073	0.001263	1.86560	14.206
4.4..	0.198064	.09691	0.005184	0.001027	1.87624	15.134
4.5..	0.188564	.09312	0.004418	0.0008331	1.88572	16.108
4.6..	0.179434	.08952	0.003760	0.0006747	1.89418	17.129
4.7..	0.170655	.08609	0.003194	0.0005451	1.90168	18.198
4.8..	0.162211	.08283	0.002709	0.0004394	1.90831	19.318
4.9..	0.154084	.07972	0.002292	0.0003532	1.91417	20.487
5.0..	0.146261	.07677	0.001935	0.0002830	1.91933	21.709
5.1..	0.138725	.07397	0.001629	0.0002260	1.92388	22.983
5.2..	0.131463	0.07130	0.001368	0.0001799	1.92784	24.312

TABLE 1—*Continued*

ξ	θ	$-\theta'$	$\theta^{3\cdot25}$	$\theta^{4\cdot25}$	$-\xi^2\theta'$	$-\frac{\xi}{3\theta'}$
5.3..	0.124461	0.06875	0.0011145	0.00001425	1.93127	25.696
5.4..	.117708	.06633	0.0009553	0.00001124	1.93427	27.136
5.5..	.111191	.06403	0.0007938	0.00008827	1.93691	28.632
5.6..	.104899	.06183	0.0006569	0.00006891	1.93911	30.188
5.7..	.098821	.05974	0.0005411	0.00005347	1.94102	31.803
5.8..	.092947	.05775	0.0004434	0.00004121	1.94264	33.479
5.9..	.087268	.05585	0.0003612	0.00003152	1.94403	35.215
6.0..	.081775	.05403	0.0002924	0.00002391	1.94519	37.014
6.1..	.076459	.05230	0.0002350	0.00001797	1.94612	38.878
6.2..	.071312	.05065	0.0001874	0.00001336	1.94691	40.805
6.3..	.066327	.04907	0.0001481	9.822×10^{-6}	1.94759	42.796
6.4..	.061496	.04756	0.00011158	7.122×10^{-6}	1.94814	44.854
6.5..	.056812	.04612	0.00008952	5.086×10^{-6}	1.94853	46.980
6.6..	.052270	.044740	0.00006828	3.569×10^{-6}	1.94887	49.173
6.7..	.047862	.043420	0.00005129	2.455×10^{-6}	1.94912	51.436
6.8..	.043584	.042157	0.00003783	1.649×10^{-6}	1.94934	53.767
6.9..	.039429	.040947	0.00002732	1.077×10^{-6}	1.94949	56.170
7.0..	.035393	.039788	0.00001923	6.806×10^{-7}	1.94961	58.644
7.1..	.031470	.038677	0.00001313	4.131×10^{-7}	1.94971	61.191
7.2..	.027656	.037611	8.626×10^{-6}	2.386×10^{-7}	1.94975	63.811
7.3..	.023947	.036588	5.402×10^{-6}	1.294×10^{-7}	1.94977	66.506
7.4..	.020337	.035606	3.176×10^{-6}	6.460×10^{-8}	1.94978	69.277
7.5..	.016824	.034663	1.715×10^{-6}	2.885×10^{-8}	1.94979	72.123
7.6..	.013403	.033757	8.1929×10^{-7}	1.098×10^{-8}	1.94980	75.046
7.7..	.010072	.032886	3.2360×10^{-7}	3.26×10^{-9}	1.94980	78.047
7.8..	.006825	.032048	9.138×10^{-8}	6.2×10^{-10}	1.94980	81.128
7.9..	.003661	.031242	1.207×10^{-8}	4×10^{-11}	1.94980	84.288
8.0..	0.000576	0.030466	2.956×10^{-11}	.....	1.94980	87.529

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