21 FUNDAMENTAL EQUATIONS OF STELLAR STRUCTURE

Mass Conservation $\frac{dM_r}{dr} = 4\pi r^2 \rho \quad or \quad \frac{dr}{dM_r} = \frac{1}{4\pi r^2 \rho}$

Hydrostatic Equilibrium $\frac{dP}{dr} = -\frac{GM_r \rho}{r^2} \quad or \quad \frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$

Energy (Thermal Equilibrium) $\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon \quad or \quad \frac{dL_r}{dM_r} = \varepsilon$

Energy Transport

RAD $\frac{dT}{dr} = -\frac{3k\rho L_r}{16\pi acr^2 T^3} \quad or \quad \frac{dT}{dM_r} = -\frac{3kL_r}{64\pi^2 acr^4 T^3}$

CONV $\frac{dT}{dr} = \frac{2}{5}\frac{T}{P}\frac{dP}{dT} \quad or \quad \frac{dT}{dM_r} = \frac{2}{5}\frac{T}{P}\frac{dP}{dM_r}$

COND $\frac{dT}{dr} = -\frac{3\kappa_c \rho L_r}{16\pi acr^2 T^3} \quad or \quad \frac{dT}{dM_r} = -\frac{3\kappa_c L_r}{64\pi^2 acr^4 T^3}$

Equation of State $P = P\left[T(r), \rho(r), \mu(r)\right]$

Opacity $\bar{k} = \bar{k}\left[T(r), \rho(r), \mu(r)\right]$

Energy Generation $\varepsilon = \varepsilon\left[T(r), \rho(r), \mu(r)\right]$

Then find $P, \rho, T, L_r, \{r \text{ or } M_r\}, \bar{k}, \varepsilon \text{ versus} \{M_r \text{ or } r\}$ with these 7 equations. Of these 4 are 1st order differential equations, so 4 boundary conditions are needed in order to solve the system of equations.

Boundary conditions:

$$at\ M_r = 0 \Rightarrow r = 0, L_r = 0$$
$$at\ M_r = M_* \Rightarrow P = 0, T = 0$$

Vogt-Russell Theorem

If P, *k*, and ε are functions only of ρ, T, and the detailed chemical composition, *the structure of the star is uniquely determined by its mass and chemical composition. (and age….)*