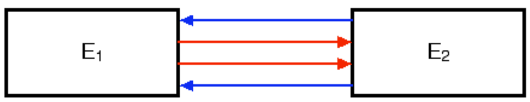
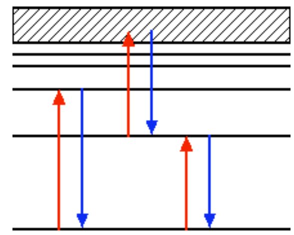
4 - Equilibrium States

1. Thermal Equilibrium

Detailed balancing for interacting systems



energy $E_1 \Rightarrow E_2$ = *energy* $E_2 \Rightarrow E_1$



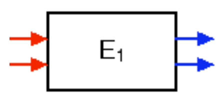
Atomic Energy Levels

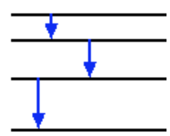
Matter particles only ⇒ "mechanical equilibrium"
Matter + Radiation ⇒ "thermodynamic equilibrium" – "TE"

In TE, all distributions are homogeneous and isotropic, and can be characterized by a single given temperature T.

2. Statistical Equilibrium

Energy In = Energy Out of a particular state





This is a less stringent condition than TE.

The type of equilibrium that exists will depend on the way that the particles in the system interact. If the *mean free path* and *mean free time* between collisions are x and t,

If the temperature is constant over:	we have:
a. times >> t and distances >> x	thermal equilibrium
b. times >> t but not distances >> x	statistical equilibrium
c. distances >> x but not times >> t	no equilibrium
d. none of the above	no equilibrium

If both matter and radiation are in thermal equilibrium (including with one another), we have TE. Sometimes, the conditions are not in "perfect TE" everywhere in the system. Nevertheless, if it is sufficiently close enough not to affect the processes sufficiently at a particular location, that location is said to be in *Local Thermodynamic Equilibrium* – LTE.

Energy Distribution of Matter and Radiation

Matter

$$n_E = \frac{2\pi n E^{1/2} e^{-E/kT}}{(\pi kT)^{3/2}} \quad (n = total\ \#\ particles\ /\ cm^3 = \int_0^\infty n_E dE)$$

Radiation

$$n_E = \frac{8\pi E^2}{h^3 c^3 \left(e^{E/kT} - 1\right)}$$

Number Density

$$n = \int_0^\infty n_z dz \quad where\ z = \nu, \lambda, E, etc.$$

$$for\ \ photons, \quad n_{total} \approx 20T^3\ cm^{-3}$$

Energy Density

$$U = \int_0^\infty U_z dz \quad where\ U_z = n_z E_z$$

Example: Energy density of Photons (set z = E)

$$U = \int_0^\infty n_E E dE = \frac{8\pi}{h^3c^3}\int_0^\infty \frac{E^3 dE}{e^{E/kT}-1}$$

$$let\ x = \frac{E}{kT} \quad dx = \frac{dE}{kT} \quad \left(i.e., E = xkT\ and\ dE = dx(kT)\right)$$

$$U = \frac{8\pi k^4 T^4}{h^3c^3}\int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$but \quad \frac{1}{e^x - 1} = \sum_{n=1}^{\infty} e^{-nx}$$

$$U = \frac{8\pi k^4 T^4}{h^3c^3}\int_0^\infty \sum_{n=1}^{\infty} x^3 e^{-nx} dx$$

$$= \frac{8\pi k^4 T^4}{h^3c^3}\sum_{n=1}^{\infty}\int_0^\infty x^3 e^{-nx} dx$$

$$but \quad \int_0^\infty x^a e^{-nx} dx = \frac{a!}{n^{a+1}}$$

$$so \quad \int_0^\infty x^3 e^{-nx} dx = \frac{3!}{n^4}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^{a+1}} = \zeta(a+1) \quad Riemann\ Zeta\ function$$

$$and\ \zeta(4) = \frac{\pi^4}{90}$$

$$\therefore \quad \sum_{n=1}^{\infty}\int_0^\infty x^3 e^{-nx} dx = \frac{3!\pi^4}{90} = \frac{\pi^4}{15}$$

$$\therefore \quad U = \frac{8\pi k^4 T^4}{h^3c^3}\frac{\pi^4}{15} = \frac{8\pi^5 k^4}{15h^3c^3}T^4 = aT^4$$

$$where\ a = \frac{8\pi^5 k^4}{15h^3c^3} = 7.56x10^{-15}\ erg\ cm^{-3}\ K^{-4}$$

$$\left(note: \quad a = \frac{4\sigma}{c}\right)$$