9 - CONTINUUM

Let us first discuss *mean free path* and *mean free time*.

Consider photons emitted at z = 0 travelling in the +z direction. Let the initial number be N_0

$$\frac{dN}{dz} = -k\rho N \quad Solution: N = N_0 e^{-k\rho z}$$

Define Mean Free Path:

$$\bar{x}_{rad} = \frac{\int_0^{N_0} z dN}{\int_0^{N_0} dN} = \frac{\int_\infty^0 z(-k\rho N dz)}{N_0}$$

$$\left(changing\ variables: N = N_0\ at\ z = 0,\ N = 0\ at\ z = \infty\right)$$

$$= \frac{\int_0^\infty k\rho(N_0)e^{-k\rho z} z\,dz}{N_0} = k\rho\int_0^\infty e^{-k\rho z} z\,dz$$

let $\tau = k\rho z, \quad d\tau = k\rho dz$

$$\bar{x}_{rad} = \frac{1}{k\rho}\int_0^\infty e^{-\tau}\tau\,d\tau = \frac{1}{k\rho}$$

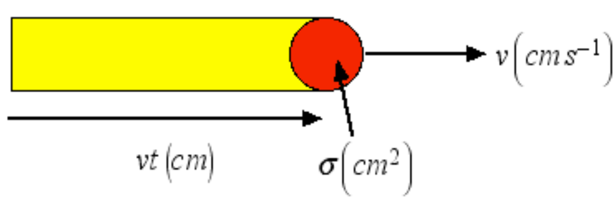
$$\left(\int_0^\infty x^{n-1}e^{-x}dx = \Gamma(n), \quad so \quad \int_0^\infty e^{-x}x dx = \Gamma(2) = \Gamma(1) = 1\right)$$

$$Mean\ Free\ Path\ for\ Radiation \quad \bar{x}_{rad} = \frac{1}{k\rho}$$

$$Mean\ Free\ Time\ for\ Radiation \quad \bar{t}_{rad} = \frac{1}{k\rho c}$$

NOTE: *1 MFP corresponds to an optical depth change $\Delta\tau$ of 1.*

For matter particles with cross section σ travelling with velocity v, in time t the particles sweep out a volume σvt cm^3 , or σv cm^3 per second.

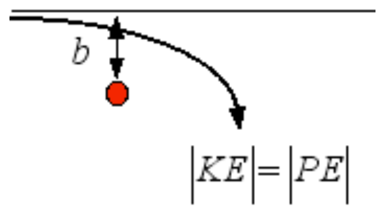


If there are n particles/cm^3, the particles will collide at a rate of $\sigma v n$ collisions/sec.

$$\text{Mean Free Path for Matter} \quad \bar{x}_{matter} = \frac{1}{\sigma n}$$

$$\text{Mean Free Time for Matter} \quad \bar{t}_{matter} = \frac{1}{\sigma n \mathrm{v}}$$

How do $\bar{x}_{rad}$ and $\bar{x}_{matter}$ compare in the Sun? First we need to estimate σ.



Let $\sigma = \pi b^2$ where b = impact parameter for $|KE| = |PE|$,

$$\frac{3}{2}kT = \frac{1}{2}m\mathrm{v}^2 = \frac{e^2}{b} \quad (\text{in cgs units, no } 1/4\pi\varepsilon)$$

$$so, \quad b = \frac{2e^2}{3kT} \quad and \quad \sigma = \frac{4\pi e^4}{9k^2T^2} \quad \left(\text{in SI} \Rightarrow b = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{3kT} \text{ and } \sigma = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi e^4}{9k^2T^2} \right)$$

In the solar photosphere,

$$k \sim 1\, cm^2/g = 0.1\, m^2/kg$$

$$n \sim 10^{17}\, cm^{-3} = 10^{23}\, m^{-3}$$

$$T \sim 6000K$$

$$\frac{\bar{x}_{rad}}{\bar{x}_{mat}} = \frac{\sigma n}{k\rho} = \frac{4\pi e^4 n}{9k_{Boltzmann}^2 T^2 k_{extinction}\rho}, \qquad but\ \rho = n\mu m_H$$

$$\frac{\bar{x}_{rad}}{\bar{x}_{mat}} = \frac{4\pi e^4 n}{9k_{Boltzmann}^2 T^2 k_{extinction} n\mu m_H} = \frac{4\pi\left(4.8x10^{-10}\right)^4}{9\left(1.38x10^{-16}\right)^2\left(6000\right)^2 \cdot 1 \cdot \left(1.67x10^{-24}\right)} > 10^{10}$$

This means that conduction is insignificant, as photons have a much larger MFP than particles. It also means that even if matter particles are in TE, the radiation may not be, if the MFP of the photons is comparable to the temperature gradient scale $T\frac{dz}{dT}$.

Is there equilibrium?

Matter

$$\bar{x}_m = \frac{1}{\rho n} = \frac{9k^2T^2}{4\pi e^4 n} = \frac{9\left(1.38x10^{-16}\right)^2\left(6000^2\right)}{4\pi\left(4.8x10^{-10}\right)^4 10^{17}}$$

$$\bar{x}_m = 10^{-4}\ cm$$

$$\bar{t}_m = \frac{1}{\sigma n v} \quad where\ \mathrm{v} \approx \sqrt{\bar{\mathrm{v}}^2} = \sqrt{\frac{3kT}{m}}$$

$$\bar{t}_m = 10^{-10}\ \sec$$

Note that this is shorter than the time between electronic transitions within atoms, so that the atomic energy levels are "thermalized".

Radiation

$$\bar{x}_{rad} = \frac{1}{k\rho} = \frac{1}{kn\mu m_H} = \frac{1}{1\cdot 10^{17}\cdot 1\cdot 1.67x10^{-24}}$$

$$\bar{x}_{rad} = 6x10^6\ cm = 60km$$

$$\bar{t}_{rad} = \frac{1}{k\rho c} = 10^{-4}\sec$$

If $T \approx constant$ over these values of $\bar{x}$ and $\bar{t}$, we have TE.

Since $\bar{x}_m \approx 10^{-4}\,cm, \bar{t}_m \approx 10^{-10}\sec$, we can be fairly certain that we have mechanical equilibrium for the matter particles.

What about the radiation? As a VERY crude estimate,

$$\left[\frac{dT}{dr}\right]_{core}^{surface} \approx \frac{T_{core,\odot}}{R_\odot} \approx \frac{2x10^7\,K}{7x10^{10}\,cm} \approx \frac{2}{3}x10^{-3}\,\frac{K}{cm}$$

$$\Delta T_{over\,MFP} = \left[\frac{dT}{dr}\right]_{core}^{surface}\cdot \bar{x}_r \sim 2000K$$

Hence, a given location "sees" photons from regions where the T's differ by over 1000K. *NOT in TE!*