Intro. Astrophysics II **NAME:**
HW #3
Due Fri. Jan. 30

FOR ALL PROBLEMS, SHOW YOUR WORK

1. Assume that the Sun is a polytrope with index n=3.25. Graph $M(r)/M_{Sun}$, T, ρ, $\log_{10}P$ versus r/R_{Sun} where appropriate (I like using atmospheres for P) using the following information:

$$\rho_c = -\frac{\xi_1}{3\left[\frac{d\theta}{d\xi}\right]_{\xi_1}} \bar{\rho}(R) \quad and \quad \rho = \rho_c \theta^n$$

$$P_c = W_n \frac{GM^2}{R^4} \quad \left(W_n = \left[4\pi(n+1)\left(\left[\frac{d\theta}{d\xi}\right]_{\xi_1}\right)^2 \right]^{-1} \right) \quad and \quad P = P_c \theta^{n+1}$$

$$T_c = \frac{P_c}{\rho_c} \frac{\mu m_H}{k} \quad and \quad T = T_c \theta$$

$$r = \left[\frac{(n+1)K}{4\pi G} \rho_c^{\frac{1-n}{n}} \right]^{1/2} \xi$$

$$M(\xi) = -4\pi \left[\frac{(n+1)K}{4\pi G} \right]^{3/2} \rho_c^{\frac{3-n}{2n}} \xi^2 \frac{d\theta}{d\xi}$$

$$K = N_n G M^{\frac{n-1}{n}} R^{\frac{3-n}{n}} \quad \underbrace{\left\{ -\frac{1}{n+1} \left[\frac{4\pi}{\xi_1^{n+1} \left(\left[\frac{d\theta}{d\xi}\right]_{\xi=\xi_1} \right)^{n-1}} \right]^{1/n} \right\}}_{"N_n"}$$

Assume a fully ionized gas with $X = 0.71, Y = 0.27, Z = 0.02$

Then compare your values with the ones for the SSM posted on the class web page. Note that I have posted many different file formats for you: MS Word, plain text, Excel, and IDL save files.

How well does the polytropic model agree with the SSM? Where they disagree, what may be the reason(s)?

WARNING: The polytropic models should agree with the SSM at r/R=0 to within about 25% or better. If they do not, you need to go back and check your work more carefully. (10 points)

HELPFUL HINTS AND SHORTCUTS

To make your life simpler and less painful, Table 4.2 of Bowers & Deeming's book, which is in the class notes, tabulates the values of $\xi_1, \frac{\rho_c}{\bar{\rho}(R)}, N_n, W_n$ for n=3.25. From these numbers you can calculate $\rho_c, P_c, T_c,$ and K fairly easily (but you need μ_{FIG} for getting T_c). Do this first, and give your results for these parameters.

Then tabulate $\theta, \theta^n, \theta^{n+1}, -\theta^{/} = -\frac{d\theta}{d\xi}, and\ -\xi^2\theta^{/}$ as a function of ξ, from ξ=0 to ξ=8.0 at intervals of 0.5 (you do not have to hand this part in). Chandrasekhar (1939, ApJ 89, p.116-118; also in the class notes), has tabulated these quantities for you.

Or if you prefer, you can calculate them yourself with a finer grid. For that, you will have to solve the Lane-Emden equation numerically. Write your own code, or find a canned program on the web to do this.

From here, you can calculate $M(\xi)/M_{Sun}, r/R_{Sun}, T, \rho, \log P \quad versus \quad \zeta$. If you work with the values from the Bowers & Deming table, tabulate your results like this:

$\zeta \quad M(\xi)/M_{Sun} \quad r(\xi)/R_{Sun} \quad T \quad \rho \quad \log P$

If you are running code, obviously the grid you use is of your choice, and may be too long to tabulate without killing too many trees; just list a short section of your output.

Although astronomers usually plot the results as a function of $M(r)/M$, for this exercise I want you to plot $M(r)/M, T, \rho,$ and $\log_{10} P$ versus r/R for your polytropic model on 4 separate plots. Overplot the SSM values to facilitate comparisons.