Decays and Decay Rates -1

- **At any time t we assume that the decay rate Γ is the same, i.e., that the same fraction of the population decays per unit time:**

$$dN(t) = -\Gamma N(t)dt$$

- **Integrating this gives**

$$N(t) = N_0 e^{-\Gamma t} = N_0 e^{-t/\tau}$$

where we have defined $\tau \equiv 1/\Gamma$.

- **We call Γ the decay rate and τ the lifetime.**

Decays and Decay Rates -2

- **What if a particle has a variety of decay modes [for example $K_S^0 \rightarrow \pi^-\pi^+$ and $K_S^0 \rightarrow \pi^0\pi^0$].**

- **If the decay rate to the first mode is Γ_1 and to the second is Γ_2 , then the probability that a particle will decay to one final state or the other in a short period of time dt is:**

$$dN = -\Gamma_1 dt - \Gamma_2 dt = -(\Gamma_1 + \Gamma_2)dt$$

from which we can conclude that the total decay rate is the sum of the partial decay rates:

$$\Gamma = \Gamma_1 + \Gamma_2 + \cdots$$

Branching Ratios

- **The relative probability that a particle decays into a particular final state is called its branching ratio or, sometimes, its branching fraction:**

$$BR_i = \Gamma_i / \Gamma_{total}$$

- **Tautologically,**

$$\sum BR_i = 1$$

- **For example,**

$$BR(K_S^0 \to \pi^- \pi^+) = (68.61 \pm 0.28)\%$$
$$BR(K_S^0 \to \pi^0 \pi^0) = (31.39 \pm 0.28)\%$$
$$BR(K_S^0 \to \pi^- \pi^+ \gamma) = (0.178 \pm 0.005)\%$$

Lifetimes and Decay Widths

- **The Heisenberg Uncertainty Principle teaches us that**

$$\Delta E \; \Delta t \sim \hbar$$

- **In its own rest frame the energy of a particle is its mass. The Heisenberg Uncertainty Principle becomes:**

$$\Delta m \; \Delta t \sim \hbar$$

- **So, quantum mechanics requires that particle lifetimes and widths (uncertainty in mass) are inversely related.**

- **The relationship we are going to derive applies to physical particles, but it also applies more generally to quantum mechanical states in all fields of physics.**

A Non-Stationary Wave Function

- **We want to consider the following wave function for a state which is not stationary:**

$$\psi(\vec{x},t) = \varphi_0(\vec{x})\; e^{-i\omega_0 t - t/2\tau}$$
$$= \varphi_0(\vec{x})\; e^{t(-i\omega_0 - \Gamma/2)}$$

- **The probability for the state to exist at time t, $t > 0$ is**

$$\psi^*\psi = \varphi^*(\vec{x}) e^{i\omega_0 t} e^{-\Gamma t/2} \varphi(\vec{x}) e^{-i\omega_0 t} e^{-\Gamma t/2}$$
$$= \varphi^*(\vec{x})\varphi(\vec{x}) e^{-i\omega_0 t} e^{-\Gamma t}$$
$$= I_0 e^{-\Gamma t} = I_0 e^{-t/\tau}$$

- **Recall that** $d\Gamma \quad |\mathrm{M}_{\mathrm{if}}|^2 \rho_f$ **and** $\Gamma = \int (d\Gamma / d\Omega)\; d\Omega$ **.**

Breit-Wigner - 1

- **In calculating M_{if} for cross-sections, we found that the Fourier transform of the spatial density distribution (the form factor)**

$$F(q^2) = \int e^{i\vec{q}\,\vec{r}} \rho(\vec{r}) d\vec{r}$$

  generally factored out of the matrix element calculation.

- **Similarly, when we calculate M_{if} for a decay process, the Fourier transform of the time dependence will appear analogously:**

$$g(\omega) = \int_0^\infty \varphi_0(\vec{x})\; e^{-i\omega_0 t} e^{-\Gamma t/2} e^{i\omega t} dt$$

$$= \int_0^\infty \varphi_0(\vec{x})\; e^{t(i[\omega_0 - \omega] - \Gamma/2)} dt$$

$$= \varphi_0(\vec{x}) \left. \frac{e^{t(i[\omega_0 - \omega] - \Gamma/2)}}{i(\omega_0 - \omega) - \Gamma/2} \right|_0^\infty$$

Breit-Wigner -2

- **Continuing the calculation:**

$$g(\omega) = \varphi_0(\vec{x}) \frac{1}{\Gamma / 2 - i(\omega_0 - \omega)}$$

- **The decay rate is proportional to the matrix element squared:**

$$|M_{if}|^2 \quad |g(\omega)|^2 \rightarrow \frac{1}{(\Gamma/2)^2 - (\omega_0 - \omega)^2}$$

$$\rightarrow \frac{1}{(\Gamma/2)^2 - (m_0 - m)^2}$$

in the decay center-of-mass.

Units

- Let's look at $|g(\omega)|^2$ with an eye toward units.

$$|\mathrm{M}_{if}|^2 \quad |g(\omega)|^2 \rightarrow \frac{1}{(\Gamma/2)^2 - (m_0 - m)^2}$$

- Γ must have units of mass as m has units of mass.
- But, from the original definition, Γ must have units of $[\text{time}]^{-1}$.
- Distance and time have units of [time].
- Mass, energy, and momentum have units of $[\text{time}]^{-1}$.
- $\hbar$ and c convert from one set of units to another

$$c = 3 \times 10^{10}\, cm/\sec; \quad \hbar c = 197\, fm - \mathrm{MeV}; \quad \hbar = 6.6 \times 10^{-22}\, \mathrm{Mev\text{-}sec}$$

Some Examples

particle (& decays)	τ (seconds)	$c\tau$ (cm)	Γ (MeV)
$\pi^+(\mu^+\nu)$	2.6×10^{-8}	780	2.5×10^{-14}
$\pi^0(\gamma\gamma)$	0.87×10^{-16}	2.6×10^{-6}	7.6×10^{-6}
$K^+\begin{pmatrix}\mu^+\nu\\ \pi^+\pi^0\end{pmatrix}$	1.2×10^{-8}	371	5.3×10^{-14}
$K_S^0\begin{pmatrix}\pi^+\pi^-\\ \pi^0\pi^0\end{pmatrix}$	0.89×10^{-10}	2.68	7.4×10^{-12}
$K_L^0\begin{pmatrix}\pi^0\pi^0\pi^0\\ \pi^-\pi^+\pi^0\\ \pi\mu\nu\\ \pi e\nu\end{pmatrix}$	5.2×10^{-8}	1554	1.3×10^{-14}