The CKM matrix elements and their determination

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LBL

Outline

- Quark mixing, C, and KM
- $|V_{ud}|$
- $|V_{cd}|$
- $|V_{us}|$
- $|V_{cs}|$
- $|V_{cb}|$
- $|V_{ub}|$
- $|V_{tb}|$
- Some known products of elements
- Parametrisations, and that phase....
- Where to from here?
- Select bibliography

Quark Mixing, C, and KM

- Cabibbo, in PRL **10** 532 (1963), started with some assumptions about J_μ, the weak current, and then wrote:
  "[We require that] J_μ has unit length. We then rewrite J_μ as
  $$J_\mu = \cos\theta(j_\mu^{(0)} + g_\mu^{(0)}) + \sin\theta(j_\mu^{(1)} + g_\mu^{(1)})"$$
  where j_μ is a vector current, g_μ is an axial-vector current, the first term is $\Delta S = 0, \Delta I = 1$, the second $\Delta S = 1, \Delta I = 1/2$.

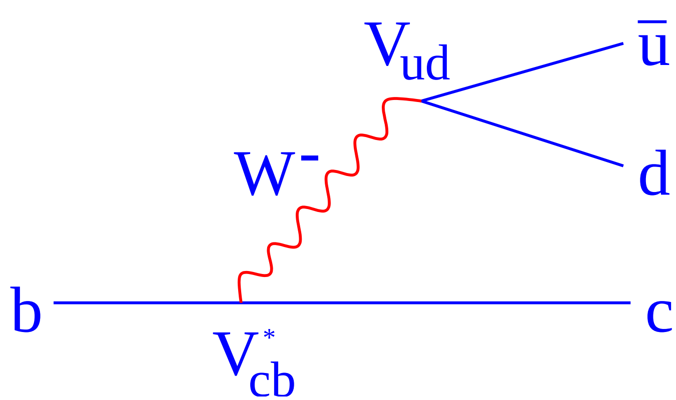
- Cabibbo noted that:
  "...the vector coupling constant for β decay is not G, but $G\cos\theta$. This gives a correction....in the right direction to eliminate the discrepancy between O^{14} and muon lifetimes."

- Ten years later, Kobayashi and Maskawa (Prog. Theor. Phys. **49** 652) expanded Cabibbo's parametrisation to a 3 generation *quark mixing matrix*

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- We now write

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Determination of the elements of V_{CKM}

- Essentially a mix of experiments of increasing difficulty moving from top left to bottom right....
- ...and some not so easy theory to cope with the fact that there's no such thing as a free quark
- 7 elements can be determined directly by experiment currently. Some information on the other two can be determined from experiments sensitive to one-loop diagrams
- More information still can be determined from CP violation experiments

Nuclear β decay and $|V_{ud}|$

- As Cabibbo noted, "all" one has to do is compare the rate of nuclear β decay to the decay rate of the muon (see Ben's talk in this series)
- Should be easy, right?
- Some general expressions:

$$ft(1 + \delta_R) = \frac{K}{G_v^2(1 + \Delta_\beta)2(1 - \delta C)} \tag{1}$$

- G_v^2 is the thing we're after $= G_\mu |V_{ud}|$. The v stands for vector - must use only $0^+ \to 0^+$ transitions so comparison to G_μ is possible.

- f is the statistical rate function:

$$f = \int_1^{E_0} pE(E_0 - E)F(Z,E)C(E)dE$$

where

  - p = mtm. of electron
  - E = energy of electron
  - E_0 maximum electron energy related to the *energy of transition*, Q_{EC} by

$$E_0 = Q_{EC}/m_e + 1$$

  - F is the Fermi function (matrix element)
  - C is a correction factor due to nuclear charge screening and dependence of F on E
- t is the partial half life - the amount of half life responsible for the $0^+ \rightarrow 0^+$ transition, corrected for the possibility of electron capture.

- Radiative corrections -
  - $\Delta\beta$ - bremsstrahlung, nucleus independent
  - δ_R - bremsstrahlung, nucleus dependent
  - δ_C - charge correction
- It turns out the last term, δ_C, is the most important. The "2" in eqn 1 assumes the nuclei are isospin singlets. They are not. More importantly, due to coulomb forces, the final state nucleus has occupies a different volume than the original and this must be corrected for. Typical values are 5–20%, with this term having the most theoretical uncertainty.
- What do experimenters measure?
  - Q_{EC} - the energy of transition
  - $\tau_{1/2}$ - the half life of the element
  - BR - the branching ratio of the $0^+ \to 0^+$ transition

  The last one is the hardest - some individual experimental errors are sometimes up to 50%. Averages are better known (the largest error is 25%.)

- 8 nuclei are used: O^{14},Al^{26}, Cl^{34}, K^{38}, Sc^{42}, V^{46}, Mn^{50} and Co^{54}.
- More information from neutron decay, but must untangle the axial and vector components. This is now under control and....
- ...finally

$$|V_{ud}| = 0.9735 \pm 0.0008$$

$|V_{cd}|$ and neutrino scattering

- Neutrino scattering off valence d quarks leads to the reactions:

$$\nu + d \rightarrow \mu^- + c$$

$$c \rightarrow s + \mu^+ + \nu_\mu$$

- This gives a characteristic di-muon signature of opposite sign
- The cross section is

$$\frac{d^2\sigma^\nu}{dx\,dy} = \frac{G^2 M E_v\, x}{\pi}\left[|V_{cd}|^2(u(x)+d(x)) + 2|V_{cs}|^2 s(x)\right]$$

$$\frac{d^2\sigma^{\overline{\nu}}}{dx\,dy} = \frac{G^2 M E_{\overline{v}}\, x}{\pi}\left[|V_{cd}|^2(\overline{u}(x)+\overline{d}(x)) + 2|V_{cs}|^2 \overline{s}(x)\right]$$

where

$$x = \frac{4E_v p_{\mu^1}\sin^2\frac{\theta}{2}}{2M_N(E_\nu - p_{\mu^1})}$$

and

$$y = \frac{E_\nu - p_{\mu^1}}{E_\nu}$$

- So (given that $s(x) - \overline{s}(x) \simeq 0$) measure

$$\frac{3}{2} B |V_{cd}|^2 = \frac{\frac{\sigma^\nu_{\mu^+\mu^-}}{\sigma^\nu_{\mu^-}} - R \frac{\sigma^{\overline{\nu}}_{\mu^+\mu^-}}{\sigma^{\overline{\nu}}_{\mu^-}}}{1 - R}$$

- Experimenters at CERN's SPS, using p beams incident on a Be target to produce neutrinos in the range 30-160 GeV.
- B is the branching fraction for $c \to l$ - estimate from D decays.

- PDG average this and a similar Tevatron experiment, to get

$$|V_{cd}| = 0.224 \pm 0.016$$

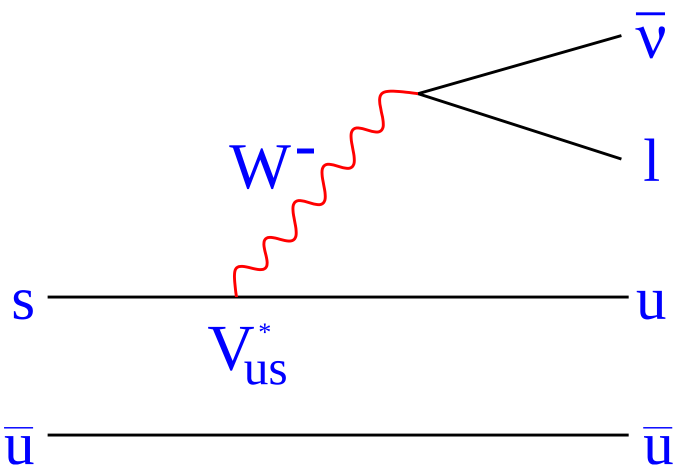
$|V_{us}|$ and the semi-leptonic decays of kaons

- The decays

$$K \to \pi^0 l\nu$$

$$K_L^0 \to \pi l\nu$$

probe the element V_{us} via the diagram



- The branching ratios are
  - $\mathrm{BR}(K^{\pm} \to \pi^0 \mu\nu_{\mu}) = 3.18 \pm 0.08 \times 10^{-2}$
  - $\mathrm{BR}(K^{\pm} \to \pi^0 e\nu_{e}) = 4.82 \pm 0.06 \times 10^{-2}$
  - $\mathrm{BR}(K^{0}{}_L \to \pi \mu\nu_{\mu}) = 38.78 \pm 0.27 \times 10^{-2}$
  - $\mathrm{BR}(K^{0}{}_L \to \pi e\nu_{e}) = 27.17 \pm 0.25 \times 10^{-2}$

- The decay is not $s \to u$ but $K \to \pi$. This is our first meeting with "hadronic uncertainties". The general expression for the decay rate is

$$\Gamma = \frac{G_F^2 m_K^5}{192\pi^3} |V_{us}|^2 C^2 |f_+(0)|^2 I (1+\delta)$$

f is one of the QCD form factors, I is a phase space integral which also is a function of these.

- The form factors are defined by the matrix element

$$\langle K, p' | \bar{u} \gamma_\mu s | \pi, p \rangle = C \left[(p'_\mu + p_\mu) f_+(t) + (p'_\mu - p_\mu) f_-(t) \right]$$

where C is a Clebsch-Gordan coefficient ($= 1/2$ for the K^+ decay and 1 for the K^0).

- To make matters worse, the integral contains the form factor $f_0(t)$ defined by

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

- The good news: the ratios

$$\bar{f}_+(t) = f_+(t)/f_+(0), \bar{f}_0(t) = f_0(t)/f_0(0)$$

(more on $f(0)$ in a minute!) are approximated well by linear parametrisations:

$$\bar{f}_+(t) = 1 + \lambda_+ t$$

with a similar expression for $f_0(t)$, and the λ parameters can be determined *by experiment*.
- The bad news: only λ_+ is well known (PDG has $\lambda_+ = 0.286 \pm 0.0022$, but $\lambda_0 = 0.006 \pm 0.007$)
- Are we stuck? No. It turns out the term in the phase space integral which needs f_0 is suppressed by m_l^2, so we can use the electron decay! Handily, this also makes getting $f_+(0)$ easier too.

- The approach to $f_+(0)$ is to treat it in an expansion

$$f_+(0) = f_0 + f_1 + f_2 + \cdots$$

where the first term has $m_u = m_d = m_s = 0$ and the rest increase the power of these masses with a fixed ratio between them.
- For $f^{K^+\pi^0}$, this creates the complication that a π looks like an η for the first term. After correcting for this, finally arrive at

$$f^{K^0\pi^+} = 0.961 \pm 0.008, f^{K^+\pi^0}/f^{K^0\pi^+} = 1.022$$

giving

$$|V_{us}| = 0.2196 \pm 0.0023$$

- Could also use $\Lambda \to pe\nu, \Sigma^- \to ne\nu$ and $\Xi^- \to \Lambda e\nu$ but the corrections from $m_u = m_d = m_s = 0$ don't agree! Not used by PDG.

$|V_{cs}|$ and neutrino scattering?

- In principle, one could use neutrino scattering off sea s quarks leads to the reactions:

$$\nu + s \rightarrow \mu^- + c$$

$$c \rightarrow s + \mu^+ + \nu_\mu$$

and indeed the experimenters who did the V_{cd} analysis published a lower limit on $|V_{cs}|$:

$$|V_{cs}| < 0.59 (90\% C.L.)$$

but why only a limit? This is the same data that has given us $|V_{ud}|$ to $< 0.1\%$.

- They found that their assumption that the strange quark sea behaved in the same way as the $\bar{u}$ and $\bar{d}$ seas was not true.

$$\frac{2S}{\overline{U} + \overline{D}} = 0.52 \pm 0.09$$

where

$$A = \int_0^1 x a(x) dx$$

- Now what? Well, can apply exactly the same arguments as for $K \to \pi l \nu$ to $D \to K l \nu$...with similar difficulties!
- The partial width is given by

$$\Gamma(D \to \overline{K} e \nu_e) \propto \left| f_+^D(0) \right|^2 |V_{cs}|^2$$

and using

$$f_+^D(0) = 0.7 \pm 0.1$$

gives

$$|V_{cs}| = 1.04 \pm 0.16$$

- Better yet (freer of hadronic uncertainties) is ALEPH's determination by measuring R_c^W,

$$R_c^W = \frac{\Gamma(W \to cX)}{\Gamma(W \to \text{hadrons})} = \frac{\Sigma_i |V_{ci}|^2}{\Sigma_i |V_{ui}|^2 |V_{ci}|^2}$$

- Using a 12 input neural network, they distinguish charm decays from everything else by exploiting charm lifetime, $D^{(*)}$ reconstruction, high energy leptons and various jet properties
- Typical efficiencies are 90% (83%) for a purity of 95% (86%) for $WW \to q\bar{q}l\nu(4q)$ events
- Result is $R_c^W = 0.51 \pm 0.05 \pm 0.03$ from which

$$|V_{cs}| = 1.00 \pm 0.11 \pm 0.07$$

- ALEPH have a result inferred from the leptonic branching fractions of the W (gives $\mathrm{B}(W \to q\bar{q})$ indirectly) which when averaged with the above is

$$|V_{cs}| = 1.034 \pm 0.051 \pm 0.029$$

This latter result is not used by the PDG.

$|V_{cb}|$ and the semileptonic decays of B mesons

- As the quarks get heavier, life gets harder
- Can't use tricks like $m_s = 0$ to untangle experimental results since m_b, m_c are large
- Go to the other extreme - treat b, c as *heavy quarks*
- Heavy quark effective theory (see Mandeep's talk next week) is a powerful tool for interpreting B physics results
- The central premise is that if m_q is large enough, the behaviour of the quark is oblivious to changes of flavour (e.g. $b \to c$).
- The result is that complex calculations can be *factorised* into long distance (peturbative) operators, and short distance (non-peturbative) ones which become tractable because of the simplifications above
- What to do?

- Inclusive spectrum

$$
\begin{aligned}
\Gamma(B \to X_q l\nu) &= \frac{G_F^2 m_b^5}{192\pi^3}\Bigg[c_3\langle B|O_3|B\rangle + c_5\frac{\langle B|O_5|B\rangle}{m_b^2} \\
&\quad + c_6\frac{\langle B|O_6|B\rangle}{m_b^3} + O(m_b^{-4})\Bigg] \\
&= \frac{G_F^2 m_b^5}{192\pi^3}\Bigg[z_0\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2}\right) - 2\left(1 - \frac{m_c^2}{m_b^2}\right)^4 \frac{\mu_G^2}{m_b^2} \\
&\quad - \frac{2\alpha_s z_0^{(1)}}{3\pi} + \cdots\Bigg]
\end{aligned}
$$

- z_0 and z_0^1 are phase space integrals. The factors μ are calculable from lattice QCD

$$
\begin{aligned}
\mu_G^2 &\simeq \frac{3}{4}(M_{B^*}^2 - M_B^2) = 0.36 \pm 0.07\text{GeV/c}^2 \\
\mu_\pi^2 &= \frac{1}{2M_B}\langle B|\bar{b}(iD)^2|b\rangle \simeq 0.5 \pm 0.2\text{GeV/c}^2
\end{aligned}
$$

- m_b has a *scale* dependence and is typically evaluated at 1 GeV

- Experimentally, what does one look for?
- First, tag a lepton with $p > 1.4$ GeV/c. Then look for another lepton. Three scenarios:
  - Second lepton was from $b \to l$. Opposite sign from the first
  - Second lepton was from $b \to c \to l$. Same sign from the first
  - Second lepton was from $b \to c \to l$ from the tag B. Opposite sign sign from the first, **BUT** likely to be back to back with the tagging lepton
- This allows the untangling of the $b \to cl\nu$ spectrum from that of $b \to c \to sl\nu$.

- The final expression for $|V_{cb}|$ for inclusive analyses is

$$|V_{cb}| = 0.0411\sqrt{\frac{B(B\to X_c l\nu)}{0.105}}\sqrt{\frac{1.55\text{ps}}{\tau_{B^0}}}\left[1-0.025\frac{\mu_\pi^2-0.5\text{GeV}^2}{0.2\text{GeV}^2}\right]$$
$$\times\left[1\pm 0.01(m_b)\pm 0.01(\text{pert.})\pm 0.015(1/m_q^3)\right]$$

which gives

$$|V_{cb}| = 40.0\pm 0.4|_{\text{exp.}}\pm 2.4|_{\text{theo.}}\times 10^{-3}$$

- Alternatively (for a long time under better theoretical control too) measure the decay rate for $B\to D^* l\nu$, and extrapolate to "zero recoil"—maximum q^2 for the lepton pair. The expression is

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3}m_{D^*}(m_B-m_{D^*})^2F^2(\omega)G(\omega)$$

where

$$\omega = \frac{(m_B^2-m_{D^*}^2-q^2)}{2m_Bm_{D^*}}$$

and

$$G(\omega) = \sqrt{\omega^2 - 1}\left[4\omega(\omega+1)\frac{1 - 2\omega m_{D^*}/m_B + m_{D^*}^2/m_B^2}{(1 - m_{D^*}/m_B)^2} + (1+\omega)^2\right]$$

- F is the (now "usual") form factor. Need to evaluate at $F(1)$ (some literature calls this $F(0)$!). Current calculations give

$$F(1) = 0.88 \pm 0.08$$

which when combined with the averages for $B(B \to D^* l\nu)$ give

$$\begin{aligned} |V_{cb}| &= 38.4 \pm 1.1 \pm 2.2 \pm 2.2 \times 10^{-3} \ \ \text{LEP} \\ &= 39.4 \pm 2.1 \pm 2.0 \pm 1.4 \times 10^{-3} \ \ \text{CLEO} \end{aligned}$$

- Overall (PDG)

$$|V_{cb}| = 0.0402 \pm 0.0019$$

$|V_{ub}|$—the same as $|V_{cb}|$?

- Following the above arguments, can't one "simply" measure the inclusive spectrum $B \longrightarrow X_u l\nu$ and/or a final state like $B \longrightarrow \rho l\nu$?
- Yes!!! But....
  - The theory is harder (u is not a heavy quark!)
  - The experiment is much harder
- *Seeing* $b \longrightarrow ul\nu$ is not so hard. The endpoint of the leptonic spectrum, beyond the kinematic limit for charm, has been observed by ARGUS and CLEO.
- But using this limited kinematic range to extract $|V_{ub}|$ is theoretically tricky

- $B \to \rho l \nu$ and $B \to \pi l \nu$ have both been seen by CLEO.
- How?
  - Neutrino reconstruction. Form

$$\begin{aligned} E_{miss} &= 2E_{beam} - \Sigma_i E_i \\ \mathbf{p}_{miss} &= -\Sigma_i \mathbf{p}_i \end{aligned}$$

    correct for splitoffs, imperfect hermiticity.
  - Ask $p_l > 1.6$GeV/c(π), $p_l > 2.0$GeV/c(ρ) to separate charm (but introduces q^2 bias - can be dealt with)
  - Use θ_{thrust} to suppress continuum. Can't use shape variables as these introduce a q^2 bias which is difficult to account for
  - Check strategy using $B \to D^* l \nu$

- Form factor calculations come from quark models or lattice QCD. Four different quark models give

$$\Gamma(B \to \rho l \nu) = 14.2, 26.1, 33.0, 11.8 \pm 3.4\text{ps}^{-1}$$
$$\Gamma(B \to \pi l \nu) = 9.6, 7.4, 7.3, 7.6 \pm 1.7\text{ps}^{-1}$$

while lattice QCD calculations give (for ρ only)

$$\Gamma(B \to \rho l \nu) = 13.8 \pm 4.0\text{ps}^{-1}$$

- Using both of their measurements for $B \to \rho l \nu$, CLEO claim

$$|V_{ub}| = 3.23 \pm 0.24^{+0.23}_{-0.26} \pm 0.58 \times 10^{-3}$$

- LEP experiments have measured the inclusive spectrum. ALEPH uses a 20 (!) variable neural network to separate the u contribution from the c contribution and quote

$$|V_{ub}| = 4.32 \pm 0.68|_{stat} \pm 0.68|_{syst} \pm 0.17|_{theo} \times 10^{-3}$$

whereas DELPHI use M_x, the mass of the system recoiling against the lepton, to determine

$$|V_{ub}| = 4.3 \pm 0.8 \times 10^{-3}$$

- None (!!!) of these results is used by the PDG, who quote only

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.090 \pm 0.025$$

($\Rightarrow |V_{ub}| = 3.6 \pm 1.0 \times 10^{-3}$). Theorists don't trust the small numbers to the right of the claims above!

$|V_{tb}|$ from top decays

- t quarks do not live long enough to form bound states
- Once you have a top decay, tagging a jet as containing a b quark will give you

$$R = \frac{B(t \to Wb)}{B(t \to WQ)} = \frac{|V_{tb}|^2}{\Sigma_i |V_{ti}|^2}$$

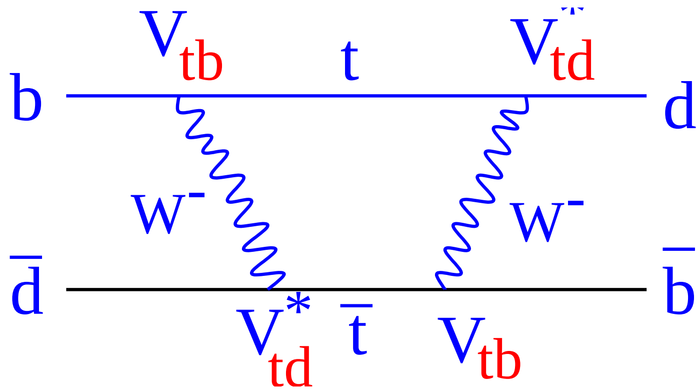
- CDF have used detached vertices and soft lepton tagging (two techniques used in their $sin2\beta$ analysis) to determine that a jet contained a b quark. Their result is

$$|V_{tb}| = 0.99 \pm 0.29$$

(note—this result was “preliminary” in 1997, and has not been updated)

Combinations: $|V_{td}^* V_{tb}|$ from B mixing

- B^0 mixing within the SM is dominated by a top quark loop



- The mixing rate is governed by ΔM_B, the difference in mass between the two states B^0_{phys} and $\overline{B}^0_{phys}$.

$$\Delta M_B = \frac{G_F^2 M_W^2 \eta_B m_B B_B f_B^2}{6\pi^2} |V_{td}^* V_{tb}|^2 S_o(x_t) e^{i\phi}$$

where

$$S_o(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}, \quad x_t = m_t^2/m_W^2$$

- The dominant uncertainty is in $B_B f_B^2$, the product of the decay constant and the "bag" parameter B. Best estimates are from lattice QCD

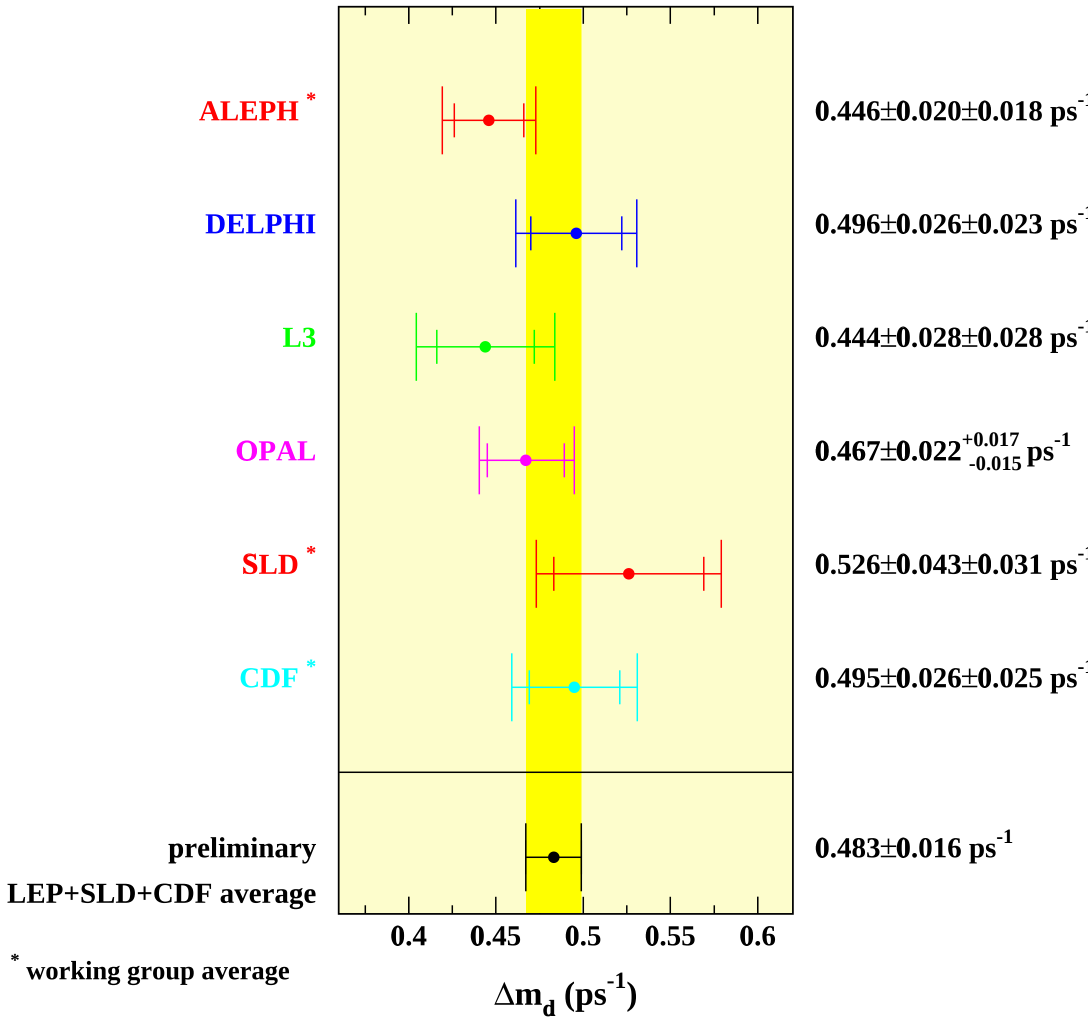
$$\sqrt{B_B} f_B = 210 \pm 40 \text{MeV}$$

- Experimentally, ΔM_B is well known

$$\Delta M_B = 0.478 \pm 0.018 \text{ps}^{-1}$$

giving

$$|V_{td}^* V_{tb}| = 0.0083 \pm 0.0016$$



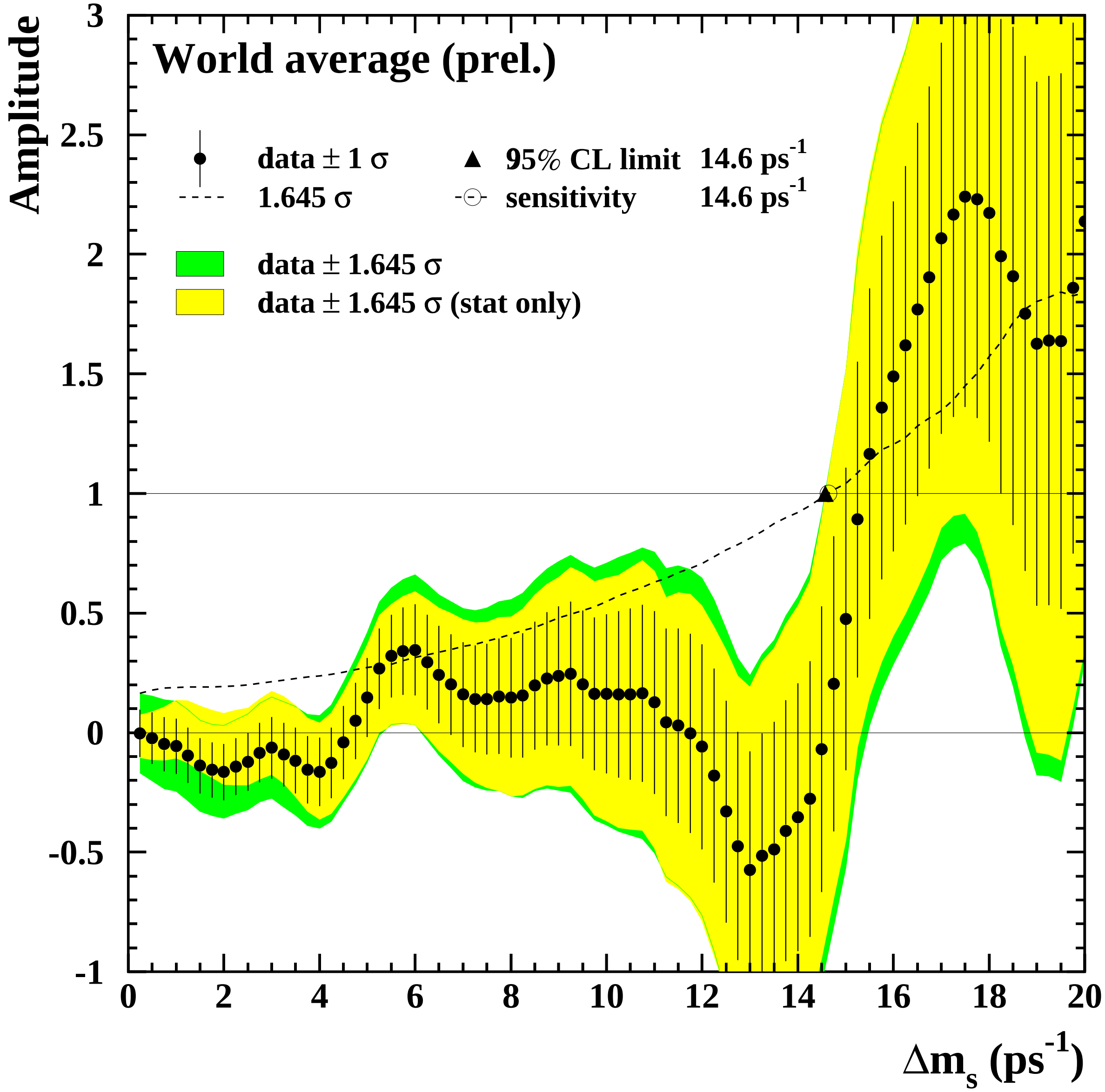
ALEPH *
0.446±0.020±0.018 ps-1
DELPHI
0.496±0.026±0.023 ps-1
L3
0.444±0.028±0.028 ps-1
OPAL
0.467±0.022 +0.017 -0.015 ps-1
SLD *
0.526±0.043±0.031 ps-1
CDF *
0.495±0.026±0.025 ps-1
preliminary
LEP+SLD+CDF average
0.483±0.016 ps-1
* working group average
0.4
0.45
0.5
0.55
0.6
Δmd (ps-1)

Combinations: $|V_{ts}^* V_{tb}|$ from B_s mixing

- The mixing of the B_s in principle gives the same information as that from B_d but the rate of oscillation is much much higher - experimentally much more difficult
- Theoretically, $B_{B_s} f_{B_s}^2$ is hard too, but the ratio

$$\left(\frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} \right)^{\frac{1}{2}} = 1.14^{+0.06}_{-0.05}$$

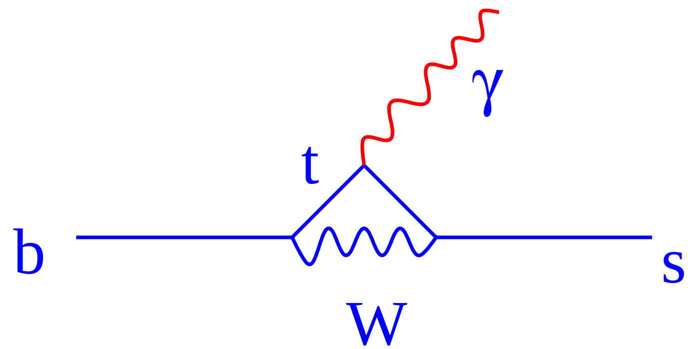
  is better known (lattice QCD again).
- Current experimental limit (combined world average) is $\Delta M_{B_s} < 14.6\text{ps}^{-1}$ (Moriond 2000).
- A word about that plot. The probability of mixing varies like $(1 - \cos \Delta_M t)$. Fit for A in $(1 - A \cos \Delta_M t)$ - a peak at 1 for a certain value of Δ_M gives the result. The A should be zero elsewhere.



World average (prel.)
Amplitude
data ± 1 σ
1.645 σ
95% CL limit 14.6 ps-1
sensitivity 14.6 ps-1
data ± 1.645 σ
data ± 1.645 σ (stat only)
Δms (ps-1)

Combinations: $|V_{ts}/V_{cb}|$ from penguin decays

- In the standard model, $b \to s\gamma$ proceeds via a penguin diagram with a top quark in the loop



- CLEO and ALEPH have performed inclusive $b \to s\gamma$ analyses, which are less prone to hadronic uncertainties than analyses like $B \to K^*\gamma$.

- The CLEO analysis looks for a high energy photon, $2.2 < E_\gamma < 2.7$ GeV, using lateral shape ID and a π^0 and η veto, then applies two techniques:
  - a neural network analysis on the shape variables of the event
  - the addition of a Kaon to the photon, plus n tracks, to make a "B".

  Both have similar sensitivity, and CLEO report

$$|V_{ts}/V_{cb}| = 1.1 \pm 0.43$$

The Unitarity Triangle, and that phase

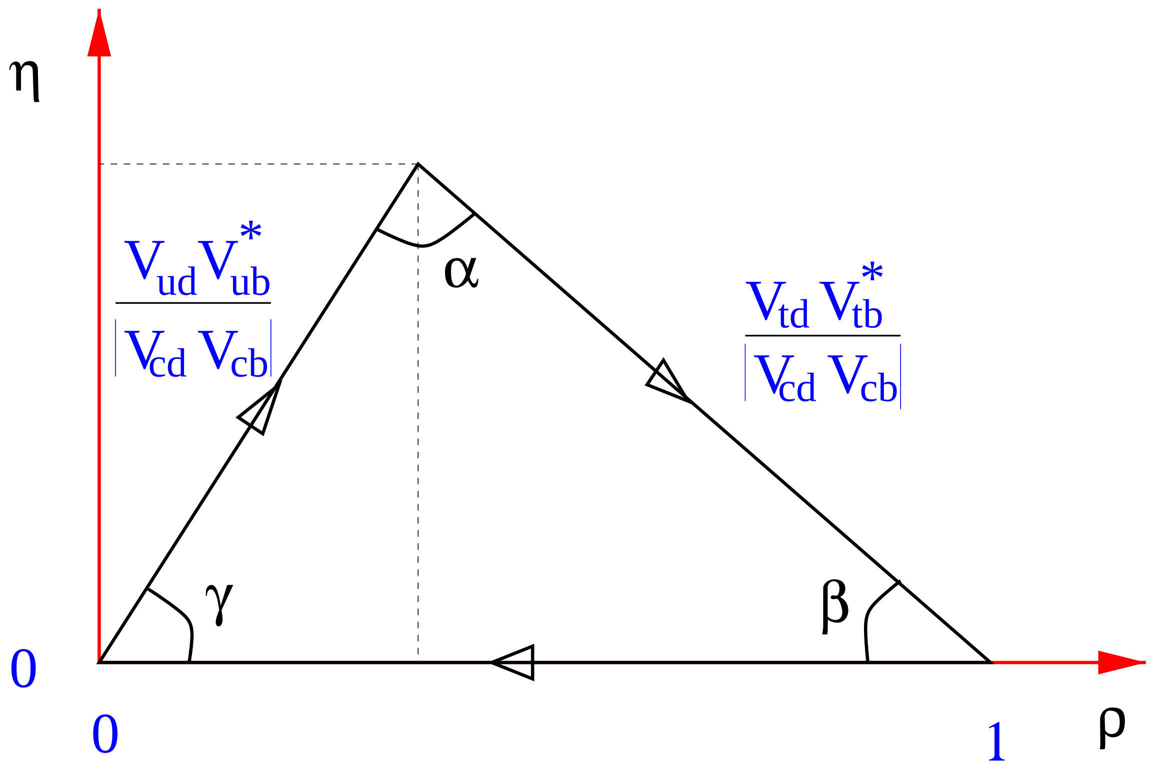
- Wolfenstein used the smallness of θ_C, the Cabibbo angle ($\cos\theta_C = V_{ud}$) to expand the matrix, setting $\lambda = \sin\theta_C$, viz:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- The Unitarity relation

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

can then be represented as a triangle in the (ρ, η) plane.



- CP violation is implied by a non-zero area for the triangle. The area is approximately equal to twice the Jarlskog invariant, J

$$J = \text{Im}(V_{\alpha j}V_{\alpha k}^* V_{\beta k}V_{\beta j}^*) = A^2\lambda^6\eta + O(\lambda^8)$$

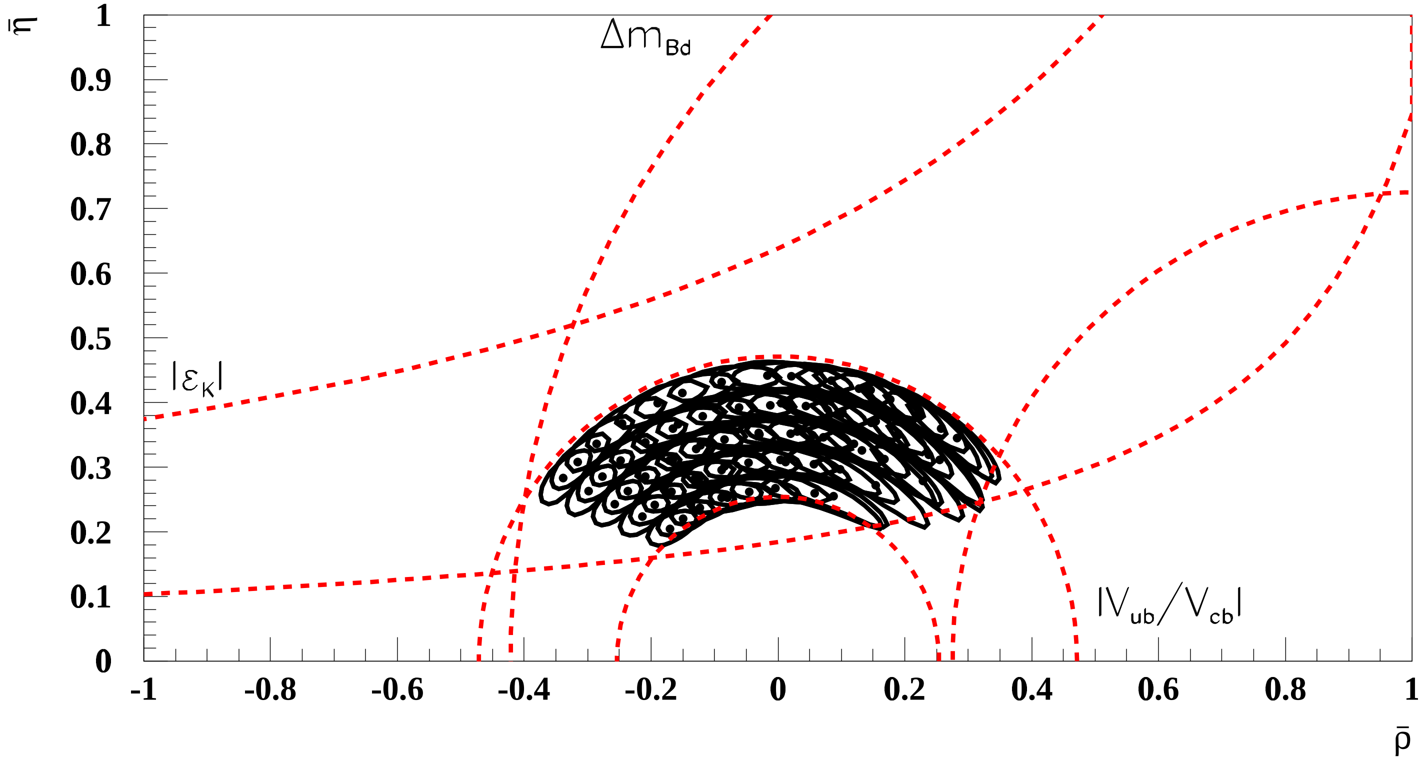
- CP violation is a small effect!

Constraints on the UT

- Several of the measurements above constrain sides of the unitarity triangle.
- In addition, the CP violation parameter ϵ_K (see Justin's talk), given by

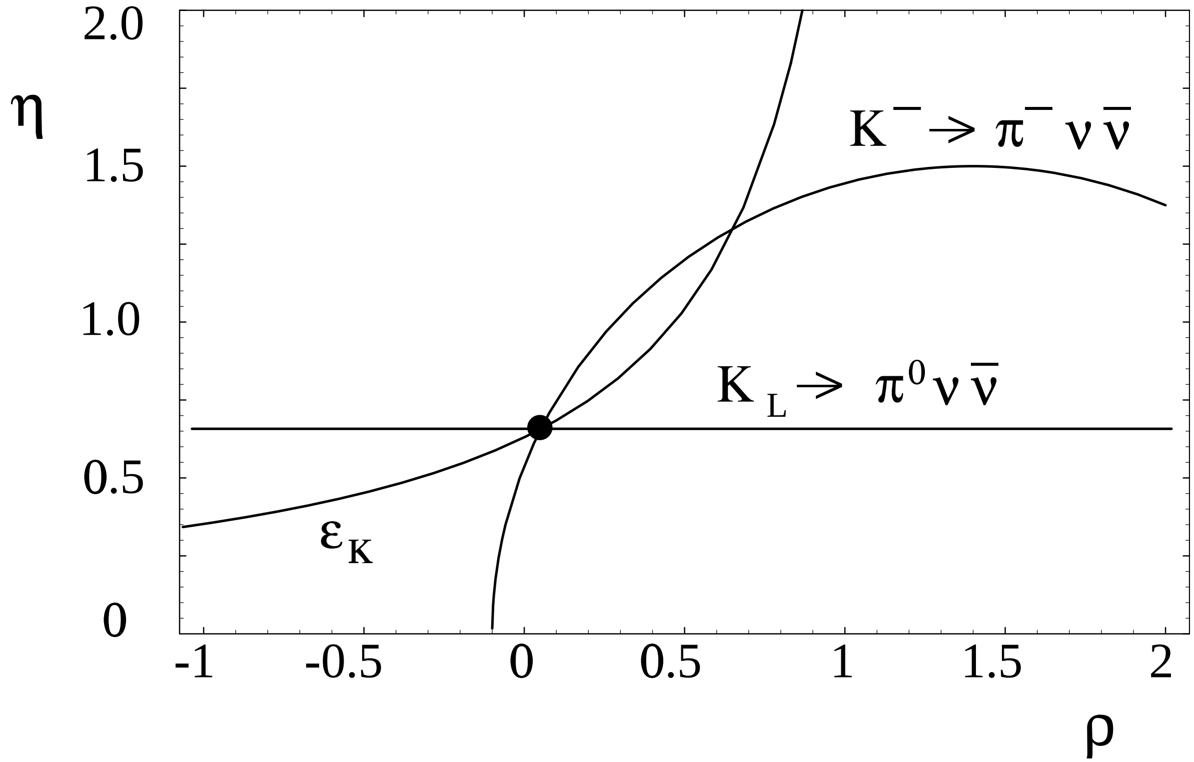
$$|\epsilon_K| = 3.8 \times 10^4 B_K A^2 \lambda^6 \eta \left[f(x_c, x_t) + A^2 \lambda^4 (1-\rho) g(x_t) \right]$$
$$= 2.258 \pm 0.018 \times 10^{-3}$$

defines a hyperbola in the (ρ, η) plane.



Where next?

- CP violation in the B system! β, α from B factories (probably in that order). γ will (probably) have to wait until LHC-B (BTeV), but let's hope not....
- (Very) rare decays of kaons



- SM predictions for the rare K decays are tiny!

$$B(K^- \to \pi^- \nu\nu) \simeq 3.4 \times 10^{-4} A^4 \lambda^{10} \left[\eta^2 + (1 + \delta_c - \rho)^2\right]$$

which is of order 10^{-11}!!

- E787 (BNL) has seen one event (in 1997)....
- ...and nothing since.

$$B(K^- \to \pi^- \nu\nu) = 1.5^{+3.4}_{-1.2} \times 10^{-10}$$

- Limit for $B(K_L^0 \to \pi^0 \nu\nu)$ (KTEV) is still 4 orders of magnitude from SM predictions.
- B_s mixing from CDF (maybe LEP/SLD)
- Further tests of HQET plus better lattice QCD should (faster computers!) allow better interpretation of B physics results
- Lots to do!

Select bibliography

- The original two
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  - M. Kobayashi and T. Maskawa, Prog. Theo. Phys **49**, 652
- All you ever wanted to know about leptonic decays of strange hadrons but were afraid to ask
  - L. Chounet *et al.*, Phys. Rep. 4 No. 5, 199
- Comprehensive notes on B physics, the CKM matrix, and much more
  - J. D. Richman, "Heavy quark physics and CP violation", Lectures given at Les Houches school, `http://hep.ucsb.edu/papers/driver_houches12.ps`
- Accessible paper on β decays
  - J. .C. Hardy *et al.* Nucl. Phys **A509** 429

- Three talks given at LP99 which explain the state of the art in b, c physics
  - A. F. Falk, hep-ph 9908520
  - G. Blaylock, hep-ex 9912038
  - R. A. Poling, hep-ex 0003025
- CLEO's $B \to \rho l \nu$ findings (in decreasing order of usefulness)
  - L. K. Gibbons, Ann. Rev. Nucl. Part. Sci. **48** 121
  - B. H. Behrens *et al.* PRD **61** 052001
  - J. P. Alexander *et al.* PRL **77** 5000
- Nice discussion of some of the theoretical uncertainties in V_{cb} and V_{ub}
  - I. I. Bigi, hep-ph 9907270
- A couple of unrelated RMP's which provide some of the background material to the later parts of this talk
  - "The discovery of the top quark", C. Campagnari and M. Franklin, Rev. Mod. Phys. **69** 137
  - "Theory of the CP-violating parameter ϵ'/ϵ", S. Bertolini and M. Fabbrichesi, Rev. Mod. Phys. **72** 65

- The neutrino experiment which extracted V_{ud}
  - H. Abramowicz *et al.*, Z. Phys. C **15** 19
- Lectures from the NATO summer school of 1998 which give very good introductions to
  - The Standard Model (M. Herrero)
  - Kaon decays (D. Bryman)
  - B physics (and a lot more) R. Aleksan
  - All in "Techniques and concepts of High Energy Physics X", ed. T. Ferbel, NATO Science Series
- And of course the PDG
  - `http://pdg.lbl.gov/1999/kmmixrpp.pdf`